PHYS 544U: Applications of E&M

- INSTRUCTOR: R. Jeffrey Wilkes
- OFFICE: B303 Physics-Astronomy Bldg
- OFFICE HOURS: after class, or by arrangement
- PHONE: 206-543-4232
- E-MAIL: wilkes@u.washington.edu

  Please send me a message from your preferred email address, so I can compile a mailing list for the class
- WEBSITE: http://www.phys.washington.edu/~jeff/courses/544U/
  - **No more paper:** after tonight, all course information and handouts will be posted on the website.
    - All students can obtain an account with UW C&C. Terminals and printers are available in Physics-Astronomy Study center (Mezzanine level of A-wing)
- Goals: In general, review and continue 543
  - review Maxwell's eqns and solutions via potentials
  - Study Special Relativity and its applications
- Homework: weekly assignments, **not** to be handed in
- Texts
  - *Feynman Lectures* vol. II (any edition)
    - Use Griffiths *Electrodynamics* as supplementary text
  - Taylor and Wheeler, *Spacetime Physics,* 2nd ed.
  - Read Einstein's original 1905 papers: go to

    http://www.fourmilab.ch/etexts/einstein/specrel/www/

    http://www.fourmilab.ch/etexts/einstein/E_mc2/e_mc2.pdf

    (links on class website)

PHYS 544: Applications of E&M

- NO EXAMS: Grade based on two Term Projects

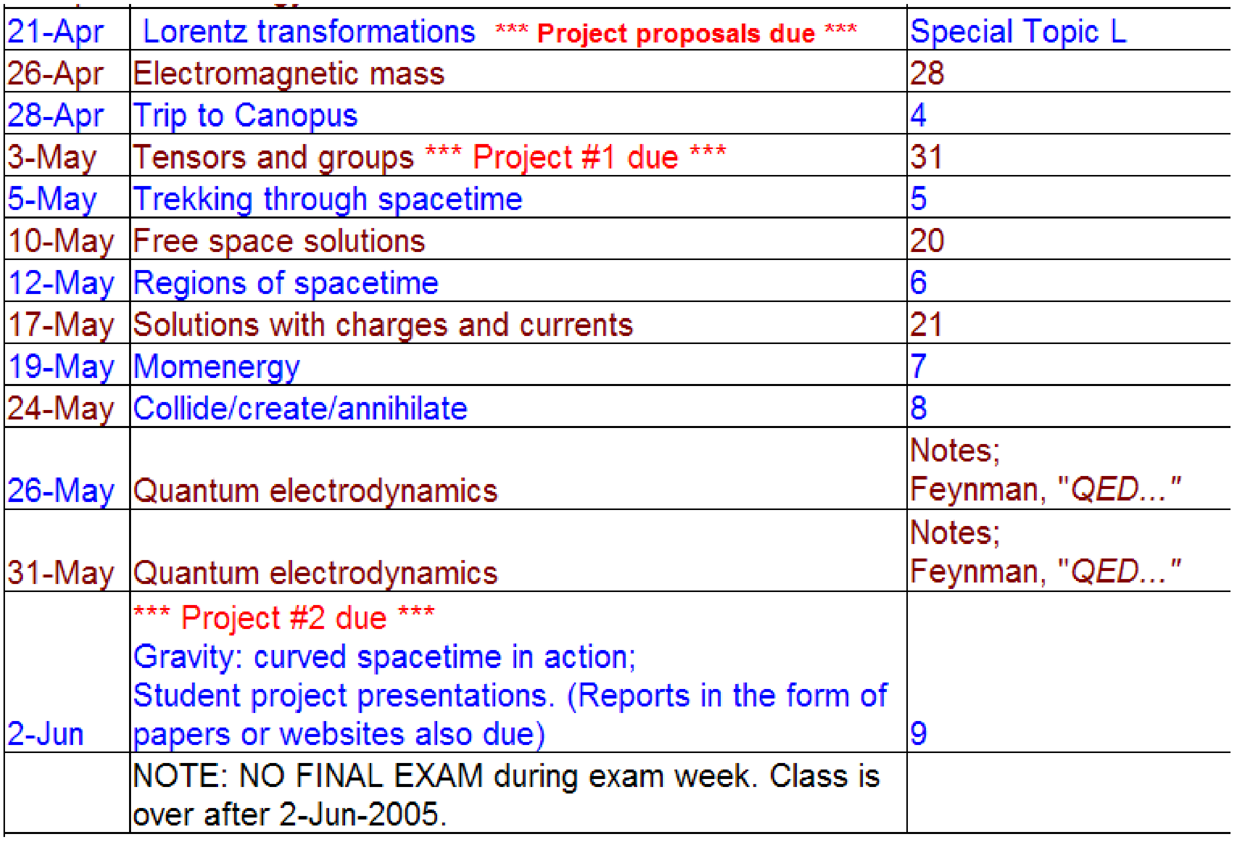
  Project 1 (due MAY 3):

  a) Find at least 10 websites devoted to Special Relativity. Select the 3 most useful sites for learning; write a 1-page summary and review for each, OR

  b) prepare a website or short (<3p) paper explaining an example or application of E&M topics covered in class.

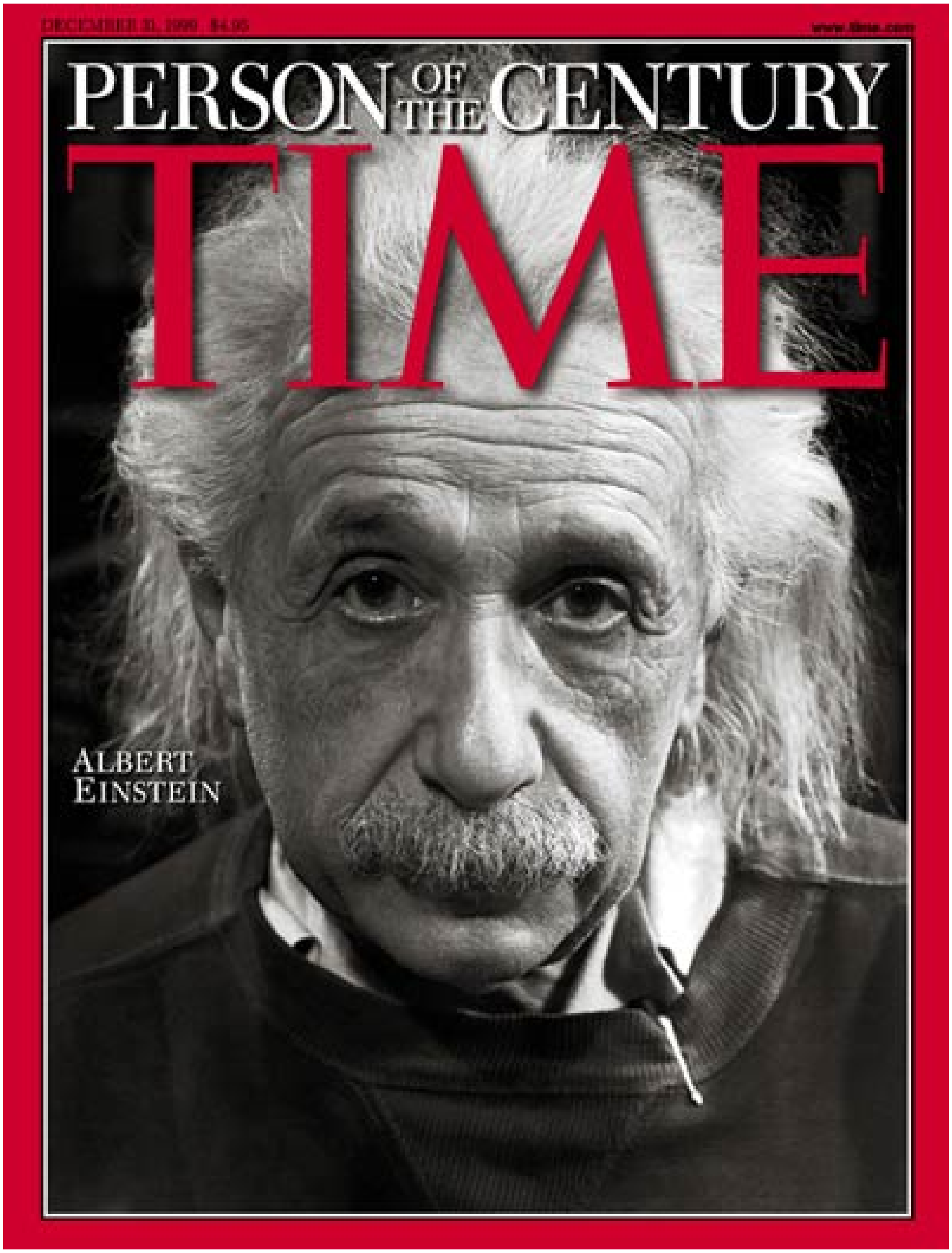
  Project 2 (due June 2): Choose a topic mentioned in the Einstein and Minkowski papers, or in Taylor & Wheeler, or in Feynman, and learn about it in greater depth than in class: read references, work out calculations, prepare demonstration graphics or software, etc.

  - Submit proposed choice of topic via email by 4/21



21-Apr	Lorentz transformations *** **Project proposals due** ***	Special Topic L
26-Apr	Electromagnetic mass	28
28-Apr	Trip to Canopus	4
3-May	Tensors and groups *** Project #1 due ***	31
5-May	Trekking through spacetime	5
10-May	Free space solutions	20
12-May	Regions of spacetime	6
17-May	Solutions with charges and currents	21
19-May	Momenergy	7
24-May	Collide/create/annihilate	8
26-May	Quantum electrodynamics	Notes; Feynman, "*QED...*"
31-May	Quantum electrodynamics	Notes; Feynman, "*QED...*"
2-Jun	*** Project #2 due *** Gravity: curved spacetime in action; Student project presentations. (Reports in the form of papers or websites also due)	9
	NOTE: NO FINAL EXAM during exam week. Class is over after 2-Jun-2005.	

Star of the course



- General plan: mostly Feynman on Tuesdays, mostly Taylor and Wheeler on Thursdays

WYP2005

World Year of
PHYSICS
2005

www.physics2005.org

From the website:

The year 2005 marks the 100th anniversary of Albert Einstein’s “miraculous year” in which he published three important papers describing ideas that have since influenced all of modern physics. This year provides the opportunity to celebrate Einstein, his great ideas, and his influence on life in the 21st century.

The US physics community’s efforts for 2005 are led by the American Physical Society, the American Association of Physics Teachers, and the American Institute of Physics, the premier organizations in the US for physicists, physics teachers, and physics societies. Our theme for the WYP celebration in the US is “Einstein in the 21st Century.”

Maxwell's equations

- Maxwell's equations ("Rationalized MKS" or SI form)

$$(1) \quad \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$(2) \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$(3) \quad \nabla \cdot \vec{B} = 0$$

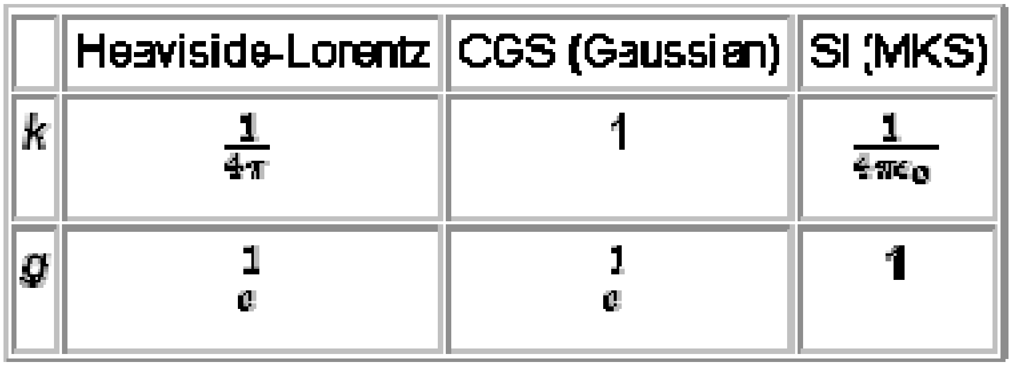
$$(4) \quad c^2 \nabla \times \vec{B} = \frac{\vec{j}}{\varepsilon_0} + \frac{\partial \vec{E}}{\partial t} \qquad \left(c^2 = \frac{1}{\mu_0 \varepsilon_0} \right)$$

  - used in Feynman, Griffiths and most freshman texts.
- All of physics as of 1890 is in Feynman's table 18-1!
- BTW: other systems of units in common use
  - CGS: used in Jackson, Ohanian and Marion.
  - Heaviside-Lorentz: used in particle physics (with c=1, so Maxwell's eqns are in their simplest possible form).

<u>CGS</u>	<u>Heaviside-Lorentz</u>
$\nabla . \mathbf{E} = 4\pi\rho$	$\nabla . \mathbf{E} = \rho$
$\nabla . \mathbf{B} = 0$	$\nabla . \mathbf{B} = 0$
$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0$	$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0$
$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{j}$	$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{c}\mathbf{j}$
$\mathbf{F} = q(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}).$	$\mathbf{F} = q(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}).$

Maxwell's equations: general format

- Define constants k and g as follows:



	Heaviside-Lorentz	CGS (Gaussian)	SI (MKS)
k	$\frac{1}{4\pi}$	1	$\frac{1}{4\pi\epsilon_0}$
g	$\frac{1}{c}$	$\frac{1}{c}$	1

- Then Maxwell's equations are given by

$$\nabla . \mathbf{E} = 4\pi k\rho,$$

also, $\mathbf{F} = k\frac{q_1 q_2}{r^2}\hat{r}$

$$\nabla \times \mathbf{E} + g\frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\mathbf{F} = q\left(\mathbf{E} + g\mathbf{v} \times \mathbf{B}\right)$$

$$\nabla . \mathbf{B} = 0,$$

$$\nabla \times \mathbf{E} + g\frac{\partial \mathbf{B}}{\partial t} = 0$$

Note: in SI units, $c^2 = \frac{1}{\mu_0\epsilon_0}$

Classical physics on one page

- The equations before Maxwell:

$$(1)\quad \nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0}$$

Gauss' Law

$$(2)\quad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

Faraday's Law

$$(3)\quad \nabla\cdot\vec{B}=0$$

(no monopoles!)

$$(4)\quad c^2\nabla\times\vec{B}=\frac{\vec{j}}{\varepsilon_0}+\cancel{\frac{\partial\vec{E}}{\partial t}}$$

Ampere's Law

$$with\quad \nabla\cdot\vec{j}=-\frac{\partial\rho}{\partial t}$$

  - Recall: these work fine for electro/magneto-*statics*

- Some basic vector and operator algebra:
  - (review Feynman Ch. 2 or Griffiths ch. 1)

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}\qquad \nabla=\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}$$

$$\nabla\cdot\nabla S=\nabla^2 S\qquad \nabla^2 S=\frac{\partial^2 S}{\partial^2 x}+\frac{\partial^2 S}{\partial^2 y}+\frac{\partial^2 s}{\partial^2 Z}\qquad \text{(Laplacian)}$$

$$(\nabla\cdot\nabla)\vec{V}=\nabla^2\vec{V}=\frac{\partial^2 v_x}{\partial^2 x}\hat{x}+\frac{\partial^2 v_y}{\partial^2 y}\hat{y}+\frac{\partial^2 v_z}{\partial^2 Z}\hat{z}\qquad \text{(vector form, Cartesian coords)}$$

$$\nabla\cdot\nabla\times\vec{V}=0\qquad and\qquad \nabla\times\nabla S=0$$

$$\nabla\times\left(\nabla\times\vec{V}\right)=\nabla\left(\nabla\cdot\vec{V}\right)-(\nabla\cdot\nabla)\vec{V}$$

$$=\nabla\left(\nabla\cdot\vec{V}\right)-\nabla^2\vec{V}\qquad \left(\text{defines } \nabla^2\vec{V},\ \text{any coords}\right)$$

$$also\oint\vec{V}\cdot dS=\int_{vol}\left(\nabla\cdot\vec{V}\right)dv\qquad \oint\vec{V}\cdot d\vec{l}=\int_{surf}\left(\nabla\times\vec{V}\right)dS$$

Integral vs differential

- Apply Green's theorem / Stokes theorem and integrate:

$$(1)\quad \nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0} \quad\rightarrow\quad \int_V \nabla\cdot\vec{E}\,dV=\oint_S \vec{E}\cdot d\vec{S}=\frac{1}{\varepsilon_0}\int_V \rho\,dV$$

$$(2)\quad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \quad\rightarrow\quad \int_S \nabla\times\vec{E}\cdot d\vec{S}=\oint_C \vec{E}\cdot d\vec{l}=-\frac{\partial}{\partial t}\int_S B\cdot d\vec{S}$$

$$(3)\quad \nabla\cdot\vec{B}=0 \quad\rightarrow\quad \int_V \nabla\cdot\vec{B}\,dV=\oint_S \vec{B}\cdot d\vec{S}=0$$

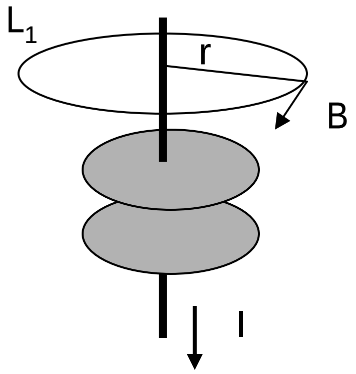
$$(4)\quad \nabla\times\vec{B}=\mu_0\vec{j}+\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \quad\rightarrow\quad \int_S \nabla\times\vec{B}\cdot d\vec{S}$$

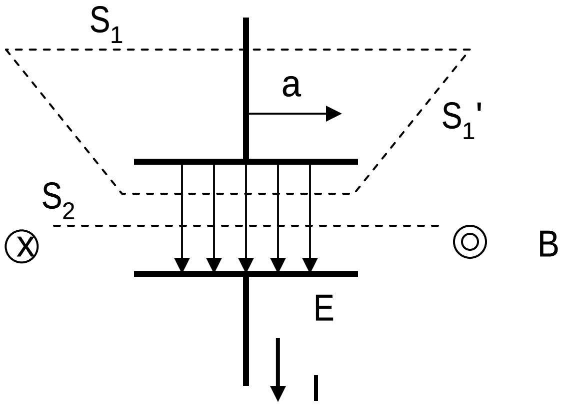
$$=\oint_C \vec{B}\cdot d\vec{l}=\mu_0\int_S \vec{j}\cdot d\vec{S}+\frac{1}{c^2}\frac{\partial}{\partial t}\int_S \vec{E}\cdot d\vec{S}$$

$$=\mu_0 I+\frac{1}{c^2}\frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\int_V \rho\,dV$$

Displacement current

- Maxwell's new term = "displacement current"
- Example: B field around wire used to charge a C





  - Charge Q on plates is changing w/time (not too quickly)

    Q(t) = I t
  - For loop L_1 , no E fields inside, but encloses I, so the integral form of (4) gives

$$c^2\nabla\times\vec{B} = \frac{\vec{j}}{\varepsilon_0} + \cancel{\frac{\partial\vec{E}}{\partial t}} \rightarrow \oint \vec{B}\cdot d\vec{l} = 2\pi r B = \frac{I_{ENCL}}{\varepsilon_0 c^2} = \mu_0 I_{ENCL}$$

  - Move L_1 down: same until we get inside plates! then I=0 and for loop L_2 we get

$$c^2\nabla\times\vec{B} = \cancel{\frac{\vec{j}}{\varepsilon_0}} + \frac{\partial\vec{E}}{\partial t} \rightarrow \oint \vec{B}\cdot d\vec{l} = \frac{1}{c^2}\frac{\partial}{\partial t}\left(\oint_{S2} EdS\right) = \varepsilon_0\mu_0\frac{\partial}{\partial t}\left(E\pi r^2\right)$$

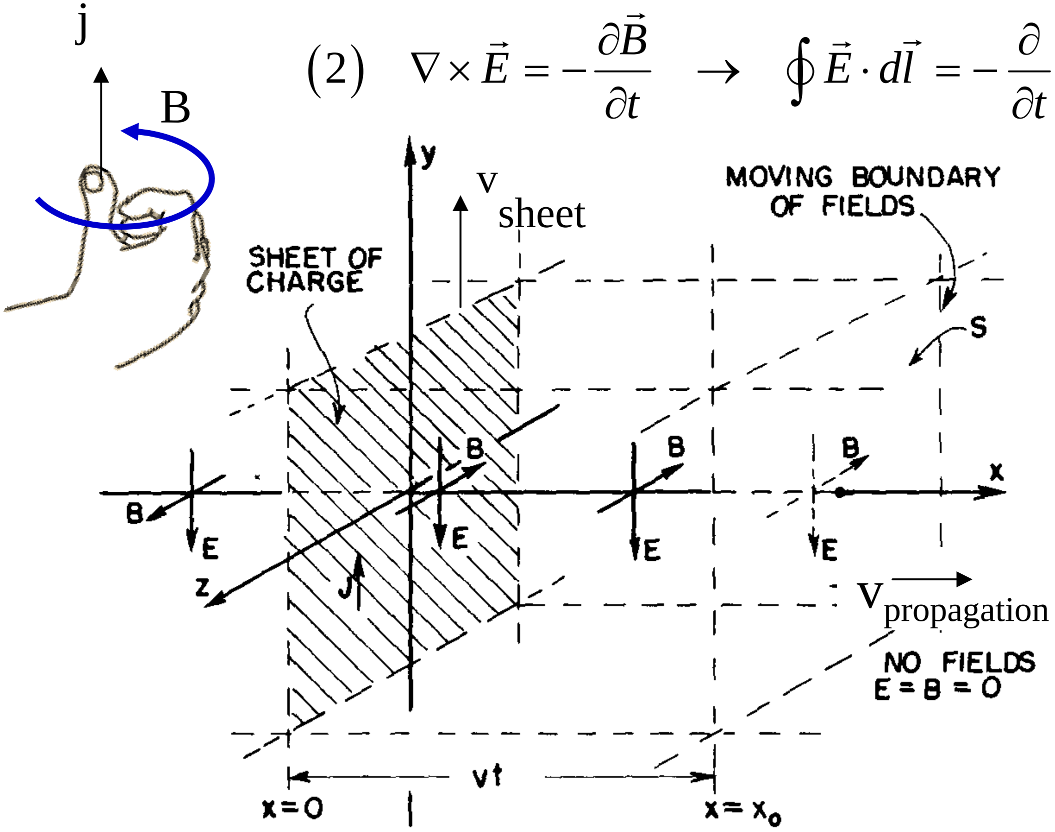
$$\text{but } (1) \rightarrow \oint_{S2} E\cdot dS = \frac{Q_{ENCL}}{\varepsilon_0} \rightarrow \vec{E} = \frac{I\,t}{\pi a^2\varepsilon_0}\hat{z}$$

$$\text{So } 2\pi r B = \varepsilon_0\mu_0\pi r^2\frac{I}{\pi a^2\varepsilon_0} \rightarrow B = \mu_0 I\frac{r^2}{a^2}$$

So what?

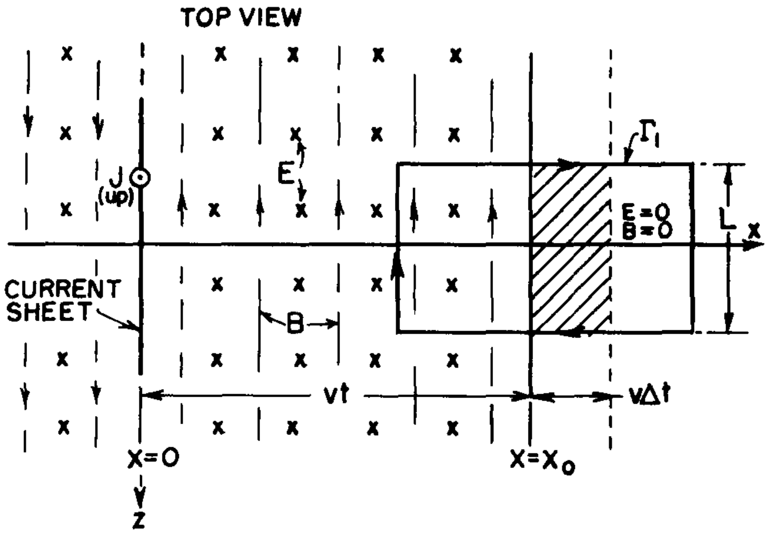
- Displacement current fixes up cases like previous example, but more importantly connects changing E and B fields
- Example: infinite current sheet at x=0
  - sheet of positive charge moving in +y direction from t=0
    - stationary negative sheet also at x=0, so no net Q
    - E=B=0 everywhere for t<0
  - For very tiny x, at t=0+dt, freshman physics problem
    - each dz of current sheet produces circular lines of B
      - add up to give B toward+z for x<0, -z for x>0
      - B still is 0 for larger |x|
    - E is in -y direction on both sides: take a loop in surface out to small x, B is changing and

$$(2)\quad \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad\rightarrow\quad \oint\vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\oint BdS$$



B,E fields *propagate*

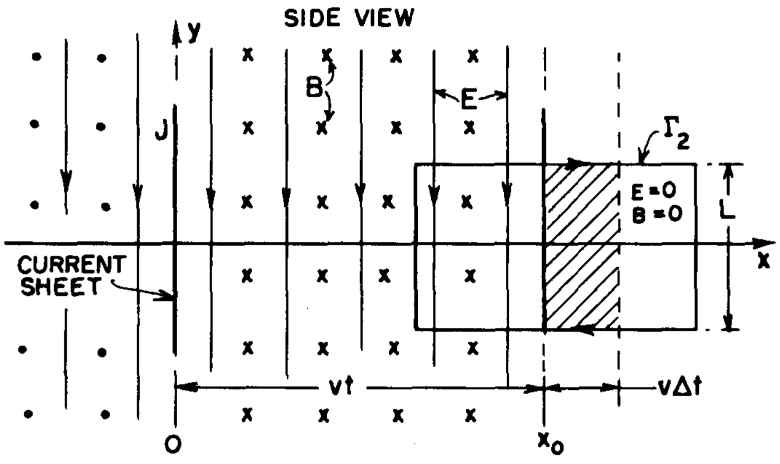
- So we have
  - E=0=B everywhere at t $\leq$ 0
  - E and B are non-zero around x=0 just after t=0
    - Can't have all of space filled instantaneously!
    - So uniform fields exist out to some x at any t
      - propagation v=x/t
  - Look at loops Γ_1 in x-z plane and Γ_2 in x-y plane, around wavefront location at time t, x=x_0:



$$\Gamma_1 : \oint_{\Gamma_1} \vec{B} \cdot d\vec{l} = \frac{1}{c^2} \frac{\partial}{\partial t} \oint_{\Gamma_1} \vec{E} \cdot d\vec{S}$$

$$\rightarrow \quad BL = \frac{1}{c^2} E \frac{\partial S}{\partial t}$$

$$\frac{\partial S}{\partial t} = vL \quad \rightarrow \quad \boxed{B = \frac{v}{c^2} E}$$



$$\Gamma_2 : \oint_{\Gamma_1} \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \oint_{\Gamma_1} \vec{B} \cdot d\vec{S}$$

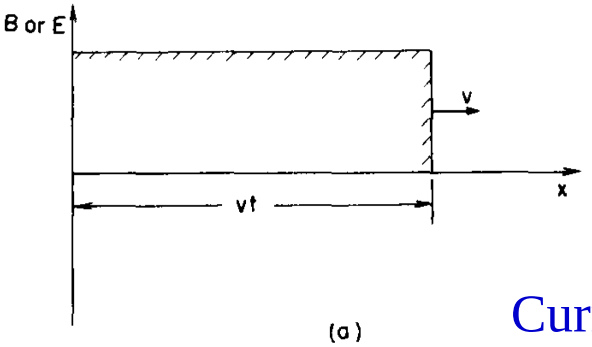
$$\rightarrow \quad EL = -B \frac{\partial S}{\partial t}$$

$$\rightarrow \quad \boxed{E = vB}$$

Can only have both results true if $v^2=c^2$: E-M fields must propagate at speed c

Propagation at speed c

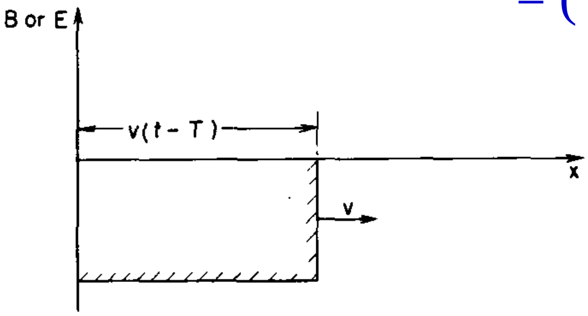
- Conclusions:
  - for t>0, E and B fields are zero beyond $x = \underline{+}\ ct$
  - E and B are perpendicular, and E = c B
  - If current sheet is stopped after time T, field cuts off at x=0 and effect also propagates outward (superposition of case a + case b below = fig c)
- New concept: B and E in pulse maintain themselves
  - independent of what happens later at x=0
    - caused by interplay of curl E (Faraday) and curl B (Maxwell displacement term) eqns



(a)

Current pulse +J for 0 < t < T
= (+J for t>0) + (-J for t>T)

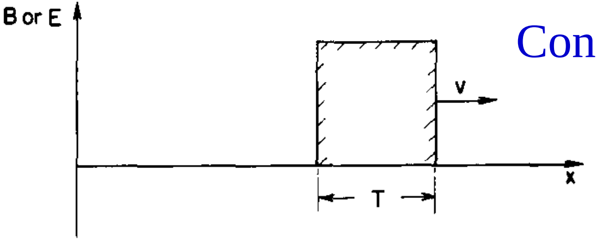
+



(b)

Pulse has a life of its own!
Continues propagating when t>T

=



(c)

A "remarkable coincidence"

- "Electrical constants" ε_0 and c appear in

$$(1) \quad \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad\qquad (4) \quad c^2 \nabla \times \vec{B} = \frac{\vec{j}}{\varepsilon_0}$$

  - We can find ε_0 by measuring force between 2 charges
  - We get $c^2\varepsilon_0$ by measuring force between 2 currents
  - Values of ε_0 and c depend on units used, but $c^2\varepsilon_0$ and ε_0 are obviously proportional: $c^2\varepsilon_0 / \varepsilon_0 = c^2$ in any units you please

- Such measurements were available to Maxwell
  - Found that the c in E&M equations ~ 3 x10^8 m/s
  - Very similar to measurements of speed of light
    - "We can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena."
    - Right (connection), and wrong (medium)!

Solving Maxwell's eqns with potentials

- First, to solve differential equations we need *boundary conditions*
  - For E&M we can take E,B → 0 at infinity, for any charge/current distribution
- Start with Maxwell (3): div B=0
  - Vector potential:

$$\nabla\cdot\left(\nabla\times\vec{V}\right)=0 \quad\to\quad \vec{B}\equiv\nabla\times\vec{A}, \quad \textit{then} \quad \nabla\cdot\vec{B}=0$$

$$\nabla\times\left(\nabla S\right)=0 \quad\to\quad \vec{A}'=\vec{A}+\nabla S \quad \textit{still gives} \quad \nabla\cdot\vec{B}=0$$

(gauge invariance)

- Next look at (2):

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}=-\frac{\partial\left(\nabla\times\vec{A}\right)}{\partial t}$$

$$\frac{\partial}{\partial x}\frac{\partial}{\partial t}=\frac{\partial}{\partial t}\frac{\partial}{\partial x} \quad\to\quad \nabla\times\left(\vec{E}+\frac{\partial\vec{A}}{\partial t}\right)=0$$

$$So \quad \vec{E}+\frac{\partial\vec{A}}{\partial t}=-\nabla\varphi \quad\to\quad \vec{E}=-\nabla\varphi-\frac{\partial\vec{A}}{\partial t}$$

  - B, E remain unchanged if we shift *both* A and ϕ (scalar potential) according to

$$\vec{A}'=\vec{A}+\nabla S \quad \textit{and} \quad \varphi'=\varphi-\frac{\partial S}{\partial t}$$

- (1) becomes $\nabla\cdot\left(-\nabla\varphi-\frac{\partial\vec{A}}{\partial t}\right)=\frac{\rho}{\varepsilon_0}$

$$So \quad -\nabla^2\varphi-\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A}\right)=\frac{\rho}{\varepsilon_0} \quad \left(\textit{potentials}\leftrightarrow\textit{sources}\right)$$

Solutions with potentials

- (4) says

$$c^2\nabla\times\vec{B}-\frac{\partial\vec{E}}{\partial t}=\frac{\vec{j}}{\varepsilon_0}\quad\rightarrow\quad c^2\nabla\times\left(\nabla\times\vec{A}\right)-\frac{\partial}{\partial t}\left(-\nabla\varphi-\frac{\partial\vec{A}}{\partial t}\right)=\frac{\vec{j}}{\varepsilon_0}$$

$$Use\qquad \nabla\times\left(\nabla\times\vec{V}\right)=\nabla\left(\nabla\cdot\vec{V}\right)-(\nabla\cdot\nabla)\vec{V}$$

$$\rightarrow\quad c^2\nabla\left(\nabla\cdot\vec{A}\right)-c^2\nabla^2\vec{A}+\frac{\partial}{\partial t}\nabla\varphi+\frac{\partial^2\vec{A}}{\partial t^2}=\frac{\vec{j}}{\varepsilon_0}$$

  - looks terrible! but we can choose the *gauge* factor
  - In statics, we set value of div A= 0, now we choose

$$\nabla\cdot\vec{A}=-\frac{1}{c^2}\frac{\partial\varphi}{\partial t}\qquad then\qquad \nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial^2 t}=-\frac{\vec{j}}{\varepsilon_0}$$

(Lorentz gauge)

$$also\quad -\nabla^2\varphi-\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A}\right)=\frac{\rho}{\varepsilon_0}\quad\rightarrow\quad\nabla^2\varphi-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial^2 t}=-\frac{\rho}{\varepsilon_0}$$

  - Same form of eqn for both vector and scalar potential!
  - If we write out the Laplacian we see

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}=-\frac{\rho}{\varepsilon_0}$$

    - Note symmetry of space and time parts

- If no sources are present, we get

$$\nabla^2\varphi-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}=0,\qquad\nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}=0$$

  - Look familiar? wave equations!

(review Feynman Sect. 20.1)

Wave solutions

- We found, for no sources, wave equations in ϕ,A:

$$\nabla^2\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = 0, \qquad \nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = 0$$

$$Then \quad \nabla\times\left(\nabla^2\vec{A}\right) = \nabla^2\left(\nabla\times A\right) \quad \left(\nabla^2 = scalar\right)$$

$$B = \nabla\times A \quad \rightarrow \nabla^2\vec{B} - \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} = 0 \quad \left(\nabla\times \Leftrightarrow \frac{\partial^2}{\partial t^2}\right)$$

$$Similarly \qquad \nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

  - Same wave equation for *fields* as well as potentials!
- Back to infinite current sheet example
  - turn current sheet on/off:
    - *pulse* of constant E and B fields propagates outward from x=0 at speed c
      - *Superposition* means we can synthesize *any* signal from this example's results
    - At any x, field *right now* depends on *past history* of current at origin: $E_y(t) \propto j_z(t - x/c)$

      t' = t - x/c is *retarded time*

$$\nabla^2 E_y - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0 \quad \rightarrow \quad E_y(t) = f\left(x - ct\right)$$

$$E_y(t) = g\left(x + ct\right)\ works\ too:\ general\ solution\ is\ E_y(t) = f + g$$

- For spherical wave, g represents a wave travelling *inward* from infinity toward origin! violates causality

f(x,t)=cos(kx)cos(kct) works as a solution - why?